An Evaluation of Input-Output Aggregation Errors Using a New MRIO Model*

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Studies of error in multi-regional input-output (MRIO) models have been focused on spatial aggregation problems on the assumption that two MRIO models developed by Polenske (1980) and Jack Faucett Associates (1983) estimate direct and indirect effects accurately. Two empirical studies based on these MRIO models reported that spatially aggregated MRIO models, consisting of a target region and others, produce small biases. However, before studying spatial aggregation errors, it was necessary to verify that MRIO-type models can be substituted for an aggregate U.S. Input-Output (USIO) model. Recently, Park et al. (2007) developed a new MRIO-type model, called the 2001 National Interstate Economic Model (NIEMO), which has 47 sectors and 52 regions (50 states, D.C. and the rest of world). For the purposes of model comparison, another 2001 spatially aggregated (USIO) model was constructed, consisting of the same 47 sectors. This one was aggregated from the IMPLAN national IO model which has 509 sectors. Because NIEMO is developed from the same data sources as USIO, comparisons among the three models are appropriate. One of the conclusions of this study is that a multiregional input-output model for the U.S., containing approximately six-million multipliers, can be constructed at low cost given the assumption that IMPLAN's outputs are plausible. And, with respect to the estimation of overall model accuracy, I found only relatively small errors when comparing the aggregates of all sectors and also only minor errors on an individual sector-by-sector comparison basis. I have also demonstrated that the sectoral aggregation required to go from IMPLAN to NIEMO imparts only minor errors. Therefore, all things considered, it is more useful to construct an MRIO-type model instead of a general one-region IO model which aggregates over economically diverse subregions.

Keywords: *Input-Output model, Multiregional Input-output model, National Interstate Economic Model, spatial and sectoral aggregation error*

I. Introduction

National economic models involve high degrees of data aggregation. It is well known that this may cause an 'ecological fallacy' as suggested by Robison (1950). For the case of national input-output models, they include both significant sectoral and spatial aggregation.

Ever since Isard (1951) proposed the "ideal" interregional input-output approach, regional scientists and others have considered ways in which spatial disaggregation could be operationalized. The various approaches to developing multi-regional input-output models (MRIOs; see Chenery (1953) and Moses (1955) for the general framework, and see Polenske (1980) and Faucett Associates (1983) for empirical applications) complicate the problem because disaggregation is obtained at the price of various simplifying assumptions that introduce further complex trade-offs for consideration.

Since Leontief (1951) had pointed to the "comparative goodness" of the sectoral aggregation errors, theoretical and empirical studies of sectoral aggregation error have been conducted with traditional regional input-output models by various scholars (Hatanaka, 1952; McManus, 1956; Theil, 1957; Ara, 1959; Morimoto, 1970; Gibbons et al, 1982; Dietzenbacher, 1992). However, as Miller and Blair (1985: p.175) noted, the spatial aggregation problem may be more difficult. Comparable interindustry matrices between regions require the same aggregated trade sectors in practical uses but generally they may rely on separate data sources and classification systems. Furthermore, complete trade data between the states are generally unavailable or difficult to use. These reasons have made empirical studies on spatial aggregation error difficult. In spite of the importance of spatial aggregation error, empirical studies for the U.S. that test the errors have been unavailable with the exception of Blair and Miller (1983) and Miller and Shao (1990).

As a new MRIO of the fifty states and the District of Columbia (and the rest of the world), I introduce the National Interstate Economic Model (NIEMO) to conduct the aggregation error tests. NIEMO is constructed from 2001 IMPLAN data sets for the 51 U.S. areas and (updated) data from the 1997 U.S. Commodity Flow Survey. To make these sources compatible, I aggregated IMPLAN's 509 sectors to 47 sectors which are labeled the "USC Sectors". Based on this effort, I examined the trade-offs involved from sectoral aggregation along with spatial disaggregation.

II. Aggregation Errors and NIEMO

Blair and Miller (1983) tested spatial aggregation error associated with an MRIO-type model based on the 1963 U.S. data constructed by Polenske (1980). Based on their three types of tests, they argue that the multiregional aggregations are "acceptable, not large", and accept the errors. These findings are consistent with their previous theoretical and hypothetical tests (Miller and Blair, 1981) for interregional aggregation error. From the results, they conclude that one can get satisfactory results from a "spatially-aggregated model" instead of a detailed NIEMO-type model (Blair and Miller, 1983: p.196). More recently, Miller and Shao (1990) reported further empirical tests for multiregional aggregation errors using the 1977 U.S. MRIO data set compiled by Jack Faucett Associates (1983).[1] They concluded that, "spatial aggregation generates less inaccuracy than sectoral aggregation." (p. 1652).

The new MRIO combines state-level input-output models from IMPLAN with interregional trade flow data from the U.S. Commodity Flow Survey (CFS) aggregated to 47 economic sectors over 52 regions (50 States, Washington, D.C., and the rest of the world), resulting in a matrix with almost 6 million cells (Park et al., 2007). Construction of the model involved substantial data assembly and considerable data manipulation, resulting in two basic tables: industrial trade coefficients tables and regional interindustry coefficients tables. While trade tables by industry are difficult to construct because of incomplete information in the CFS data, the interindustry tables present no serious problems because reliable data are available from IMPLAN at the state and industry levels. For the commodity trade flows, this study adopts a new approach initially developed by Park et al. (2004) for estimating interstate trade flows for non-service sectors, based on the 1997 Commodity Flow Survey data via an AFM (adjusted flow model) and a DFM (doubly-constrained Fratar model).[2]

The initial version of NIEMO was developed as a demand-side model and was used to estimate the economic impacts of hypothetical terrorist attacks on major three ports of U.S. (Park et al., 2007) and the economic impacts due to the mad-cow-disease outbreak in the state of

[1] Actually, Miller and Shao used data updated and reported in 1988 by various Boston College researchers.
[2] Very recently, Freight Analysis Framework (FAF) released the 2002 FAF[2] commodity origin-destination database (http://ops.fhwa.dot.gov/freight/freight_analysis/faf/faf2_tech_document.htm, 2006). Because the trade data for commodity sectors include service values used up in commodity processing and shipping, however, it is hard to say that the values released are comparable to CFS commodity flows.

Washington (Park et al., 2006).[3] Although the latter tested sectoral aggregation from 509 IMPLAN sectors to 47 USC sectors and revealed only minor errors in the sectoral aggregation, it still leaves out thorough tests of whether NIEMO can be substituted for IMPLAN or not.

This study, therefore, involves a comparison of sectoral and spatial aggregation, because NIEMO was aggregated sectorally but disaggregated spatially. To focus on the trade-offs between NIEMO's sectoral aggregation and spatial disaggregation, this study of aggregation and disaggregation tests relies upon a different approach than in the previous studies of aggregation error tests. This study starts assuming an ideal interstate input-output model.

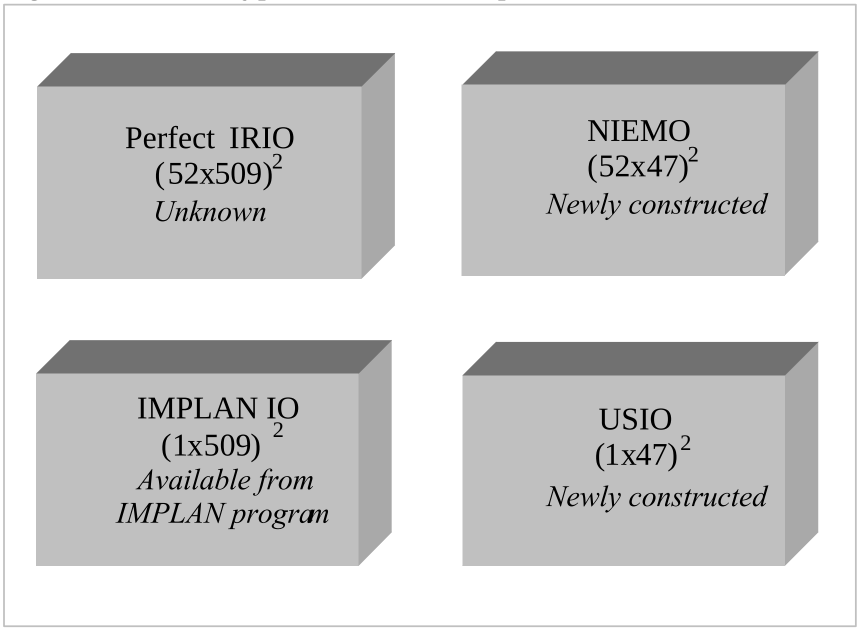
III. Data and Model

My approach depends on 2001 IMPLAN data. Before testing the sectoral and spatial aggregation error tests, tests of data compatibility are required. First, two types of Input Output model with 47 USC sectors at the national level, USIO and USIO_U, are developed. The former is a traditional Leontief industry-by-industry type IO for the aggregation error tests, and the latter is a sectoral aggregation IO model, the "use matrix" of IMPLAN. The USIO_U is used to verify the compatibility of IMPLAN data and the newly constructed USIO and NIEMO for the aggregation and disaggregation tests. Because the IMPLAN industry total outputs are based on a use matrix, the results via USIO_U should be the same as the IMPLAN industry total outputs when value-added data are input.

Figure 1 shows the plan of this study. With three models in hand, it is possible to estimate errors of industrial and spatial aggregation so that we can learn about the trade-offs and possible benefits when moving from a spatially aggregated input-output model with considerable sectoral detail to a spatially disaggregated model that has less sectoral detail. In Figure 1, the upper-left box indicates the unknown "perfect" interregional input-output model, IRIO, containing 509 sectors and 52 regions. Just below it and to the right is NIEMO, aggregated to 47 sectors. In the lower-left is the IMPLAN model for the U.S., but with 509 sectors. In the lower right is an aggregation of IMPLAN sectors to the 47 USC sectors used in NIEMO, which is called USIO.

[3] A supply-side NIEMO was developed recently by Park (2006a), and used to test the hypothetical economic impacts of closures of the twin ports of Los Angeles and Long Beach due to 'dirty-bomb' attacks.

Figure 1. Various types of IO models specified



Further, instead of BEA GSP (Gross State Product) data, which are some of the most credible data, IMPLAN value added data were used as the exogenous inputs for two reasons. First, IMPLAN data reveal greater accuracy when the 51 areas are aggregated to the U.S. than the BEA GSP data, as shown in Appendix 1. Also, The BEA GSP data do not include total output for each state and therefore NIEMO cannot input the value added vector for every state. In Appendix 2, there is a comparison table of total output between IMPLAN and BEA data and between the sum of states for IMPLAN and national IMPLAN for 47 USC sectors.

To address the issue of data compatibility, a transposed IO matrix of $\mathbf{A}$ is first suggested as in equations (1) to (4).

$$\mathbf{X} = \mathbf{X}\mathbf{A}^{\mathbf{T}} + \mathbf{V} \tag{1}$$

where, total output row vector $\mathbf{X} = [x_1, \cdots, x_n]$, total value-added row vector $\mathbf{V} = [v_1, \cdots, v_n]$, and technical coefficients matrix $\mathbf{A} = \mathbf{Z}\hat{\mathbf{X}}^{-1}$ when there are n industry sectors, where matrix

$\mathbf{Z} = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix}$ is a matrix of the element z_{ij}, denoting the deliveries in dollars from industry sector i to j.[4]

Hence, to estimate new total output based on current value added, equation (1) converts to equation (2), and shown as,

$$\mathbf{X} = \mathbf{V}(\mathbf{I} - \mathbf{A}^{\mathbf{T}})^{-1} \quad (2)$$

Similar to equation (1), the technical coefficients matrix of USIO_U, $\mathbf{U}^{\mathbf{A}}$, is constructed based on the use matrix, $\mathbf{U}$, shown as equation (3),

$$\mathbf{X}^{\mathbf{S}} = \mathbf{X}^{\mathbf{S}}(\mathbf{U}^{\mathbf{A}})^{\mathbf{T}} + \mathbf{V} \quad (3)$$

where $\mathbf{U}^{\mathbf{A}} = \mathbf{U}(\hat{\mathbf{X}}^{\mathbf{S}})^{-1}$. Note $\mathbf{U}$ is a use matrix from IMPLAN data, where $\mathbf{X}^{\mathbf{S}}$ a vector of IMPLAN industry total outputs, that is $x_j^s = \sum_i u_{ij}$. u_{ij} denotes deliveries in dollar values from input commodity i to industry sector j.

Therefore, the total outputs of USIO_U are estimated as,

$$\mathbf{X}^{\mathbf{S}} = \mathbf{V}[\mathbf{I} - (\mathbf{U}^{\mathbf{A}})^{\mathbf{T}}]^{-1} \quad (4)$$

[4] The hat of $\hat{\mathbf{X}}$ notes a diagonalized matrix from a vector $\mathbf{X}$.

Table 1. Model estimates of NIEMO and two USIOs using value added and error comparisons with aggregations of IMPLAN and BEA total output (units: $m.)

	Total Output					Aggregation Errors		
USC Sec.	NIEMO[1]	USIO[2]	USIO_U[2]	IMPLAN[3]	BEA[4]	ANIEMO[5]	AUSIO[6]	AUSIO_U[7]
USC 1	153,313	150,992	172,661	173,097	195,922	-11.43%	-12.77%	-0.25%
USC 2	110,960	108,985	118,664	118,853	117,442	-6.64%	-8.30%	-0.16%
USC 3	39,587	39,741	44,678	44,785	50,631	-11.61%	-11.26%	-0.24%
USC 4	79,370	78,920	84,758	84,932	105,455	-6.55%	-7.08%	-0.20%
USC 5	258,648	255,814	285,412	286,070	309,327	-9.59%	-10.58%	-0.23%
USC 6	55,643	53,332	61,415	61,546	42,350	-9.59%	-13.35%	-0.21%
USC 7	51,122	49,874	52,563	52,637	17,100	-2.88%	-5.25%	-0.14%
USC 8	17,923	17,410	19,023	19,049	18,364	-5.91%	-8.60%	-0.13%
USC 9	8,359	7,825	9,115	9,129	8,813	-8.43%	-14.28%	-0.15%
USC 10	310,773	276,341	370,696	371,603	376,181	-16.37%	-25.64%	-0.24%
USC 11	67,975	64,587	75,862	76,034	75,787	-10.60%	-15.06%	-0.23%
USC 12	126,365	120,135	134,193	134,457	134,295	-6.02%	-10.65%	-0.20%
USC 13	14,712	13,864	16,177	16,209	16,157	-9.24%	-14.47%	-0.20%
USC 14	128,487	122,630	141,830	142,133	142,483	-9.60%	-13.72%	-0.21%
USC 15	182,180	173,396	203,203	203,666	200,586	-10.55%	-14.86%	-0.23%
USC 16	91,709	87,159	101,490	101,676	103,276	-9.80%	-14.28%	-0.18%
USC 17	127,697	120,112	142,028	142,353	141,106	-10.30%	-15.62%	-0.23%
USC 18	193,074	188,166	203,515	203,883	213,869	-5.30%	-7.71%	-0.18%
USC 19	148,667	145,164	172,635	172,998	130,168	-14.06%	-16.09%	-0.21%
USC 20	90,946	88,630	97,633	97,801	97,201	-7.01%	-9.38%	-0.17%
USC 21	102,573	91,573	121,217	121,498	118,894	-15.58%	-24.63%	-0.23%
USC 22	165,613	159,533	184,164	184,519	182,176	-10.25%	-13.54%	-0.19%
USC 23	294,577	281,213	330,681	331,350	322,670	-11.10%	-15.13%	-0.20%
USC 24	542,474	510,070	600,089	601,195	608,161	-9.77%	-15.16%	-0.18%
USC 25	373,244	354,808	445,979	447,184	441,871	-16.53%	-20.66%	-0.27%
USC 26	100,183	92,634	117,725	118,010	118,170	-15.11%	-21.50%	-0.24%
USC 27	103,896	97,461	113,919	114,130	112,783	-8.97%	-14.60%	-0.18%
USC 28	66,855	64,792	73,493	73,637	73,417	-9.21%	-12.01%	-0.19%
USC 29	99,929	91,526	105,745	105,923	105,886	-5.66%	-13.59%	-0.17%
USC 30	268,644	254,021	296,155	296,699	343,430	-9.46%	-14.38%	-0.18%
USC 31	937,997	930,238	1,011,034	1,013,11	899,778	-7.41%	-8.18%	-0.21%
USC 32	857,192	844,892	873,626	875,258	851,286	-2.06%	-3.47%	-0.19%
USC 33	473,816	459,999	501,917	502,771	469,464	-5.76%	-8.51%	-0.17%
USC 34	157,525	155,967	162,091	162,269	101,953	-2.92%	-3.88%	-0.11%
USC 35	926,980	918,234	941,151	942,803	1,021,03	-1.68%	-2.61%	-0.18%
USC 36	562,979	553,430	585,257	586,269	710,579	-3.97%	-5.60%	-0.17%
USC 37	1,275,606	1,261,62	1,285,741	1,287,27	1,361,67	-0.91%	-1.99%	-0.12%
USC 38	1,661,115	1,650,56	1,679,422	1,681,50	1,775,39	-1.21%	-1.84%	-0.12%
USC 39	994,981	988,131	1,006,962	1,008,25	1,105,60	-1.32%	-2.00%	-0.13%
USC 40	207,801	206,390	209,967	210,209	290,414	-1.15%	-1.82%	-0.12%
USC 41	435,826	432,495	443,272	443,881	481,024	-1.81%	-2.57%	-0.14%
USC 42	84,076	83,462	85,545	85,680	107,147	-1.87%	-2.59%	-0.16%
USC 43	1,147,180	1,133,27	1,186,485	1,188,87	1,094,72	-3.51%	-4.68%	-0.20%
USC 44	151,333	150,017	154,024	154,279	154,140	-1.91%	-2.76%	-0.17%
USC 45	475,583	470,460	497,804	498,852	500,925	-4.66%	-5.69%	-0.21%
USC 46	1,279,201	1,276,54	1,288,675	1,288,98	2,019,17	-0.76%	-0.96%	-0.02%
USC 47	704,222	691,941	754,487	755,883	534,941	-6.83%	-8.46%	-0.18%
TOTAL	16,708,912	16,368,3	17,564,17	17,593,2	18,403,2	-5.03%	-6.96%	-0.16%

Note: 1. After being estimated by NIEMO, total outputs are aggregated spatially from 51 states to U.S.

2. Two types of USIOs consisting of 47 USC sectors and U.S. are newly constructed

3. 509 IMPLAN total outputs in use matrix are simply aggregated to 47 USC sectors using an IMPLAN_USC bridge table, where I dropped IMPLAN 5 sectors, or 501-503 and 507-508, due to nonexistence of coefficients for those sectors.

4. NAICS sectors of BEA suggested in ‘http://www.bea.doc.gov/bea/industry/gpotables/gpo_action.cfm?anon=315&table_id=10984&format_type=0’ are aggregated to 47 USC sectors

5. ANIEMO=(NIEMO-IMPLAN)/IMPLAN

6. AUSIO =(USIO -IMPLAN)/IMPLAN

7. AUSIO_U=(USIO_U-IMPLAN)/IMPLAN

Two USIO models used the aggregated value-added vector of 47 USC sectors from IMPLAN's 509 sectors; those estimates via equations (2) and (4) are compared with the actual 509 IMPLAN industry total outputs aggregated to 47 USC sectors. The third through fifth columns in Table 1 show comparisons between the estimated results via USIO and USIO_U and the aggregated IMPLAN industry total outputs. Similarly, for the use of aggregation and disaggregation tests, the total output of the demand-driven NIEMO via the transposed matrix of $\mathbf{W}\,(=\mathbf{CA})$ will provide useful comparisons with those aggregate models and the aggregate IMPLAN industry total output. The equation is shown in (5), where new total outputs $\mathbf{X_N}$ via NIEMO based on the value added vector for 51 states are summed spatially to the U.S.

$$\mathbf{X_N} = \mathbf{V_N}(\mathbf{I} - \mathbf{W^T})^{-1} \tag{5}$$

where, $\mathbf{X_N}$ and $\mathbf{V_N}$ is a (52x47)x1 row vector, and $\mathbf{W^T}$ (52x47)x(52x47) is the transposed matrix of $\mathbf{W}$, where $\mathbf{W} = \mathbf{CA}$ described in the NIEMO construction discussion.[5] Note matrix $\mathbf{C}$ is a (52x47)x(52x47) diagonal block matrix of interstate trade flow of each USC industry sector and $\mathbf{A}$ is a (52x47)x(52x47) block diagonal interindustry matrix of each state.

Three estimated results and aggregated IMPLAN total outputs are shown in Table 1, where all USC sectors via USIO_U do not show more than 0.3 percentage sectoral aggregation errors from the aggregated IMPLAN total output.[6] However, because the aggregation errors by total output via USIO stem from adjusted coefficients using make and use tables, it is hard to say that there are sizable sectoral aggregation errors from those USIO models built with the IMPLAN data.

Similarly, the total outputs via NIEMO have aggregation and disaggregation errors when comparing the aggregated IMPLAN data. However, this includes the errors when combining inter-industry matrices with trade matrices. Therefore, apart from the direct comparison of IMPLAN industry total output, it is still necessary to estimate new total outputs via the IMPLAN

[5] However, all USC sectors in the Rest of World (=1x47) are zero elements, because there is no valued-added information for this region.
[6] The definitions of 47 USC sectors are shown in Appendix 3.

IO model in order to compare sectoral aggregation and spatial disaggregation errors between NIEMO and the IMPLAN IO.

Nevertheless, because these small errors via USIO_U denote that USIO and NIEMO are constructed based on plausible bridges and aggregation processes[7], this study invoked further tests, by estimating new total outputs with new final demands for the demand-side IMPLAN IO, USIO, and NIEMO models. These models are expressed as,

$$\mathbf{X}_m = (\mathbf{I} - \mathbf{B}_m)^{-1} \mathbf{\Gamma}_m \tag{6}$$

where, $\mathbf{X}_m$ is total output vector via IO model m, which denotes a type of IO model, IMPLAN IO (I), USIO (UI or UN), or NIEMO (N),

$\mathbf{B}_m$ is the coefficients matrix for IO model m, where $\mathbf{B}$ is a 509x509 industry-by-industry technical IO coefficients if m is IMPLAN IO model, a 47x47 aggregated industry-by-industry technical IO coefficients if m is USIO model, or a (52x47)x(52x47) NIEMO coefficients if m is NIEMO, and

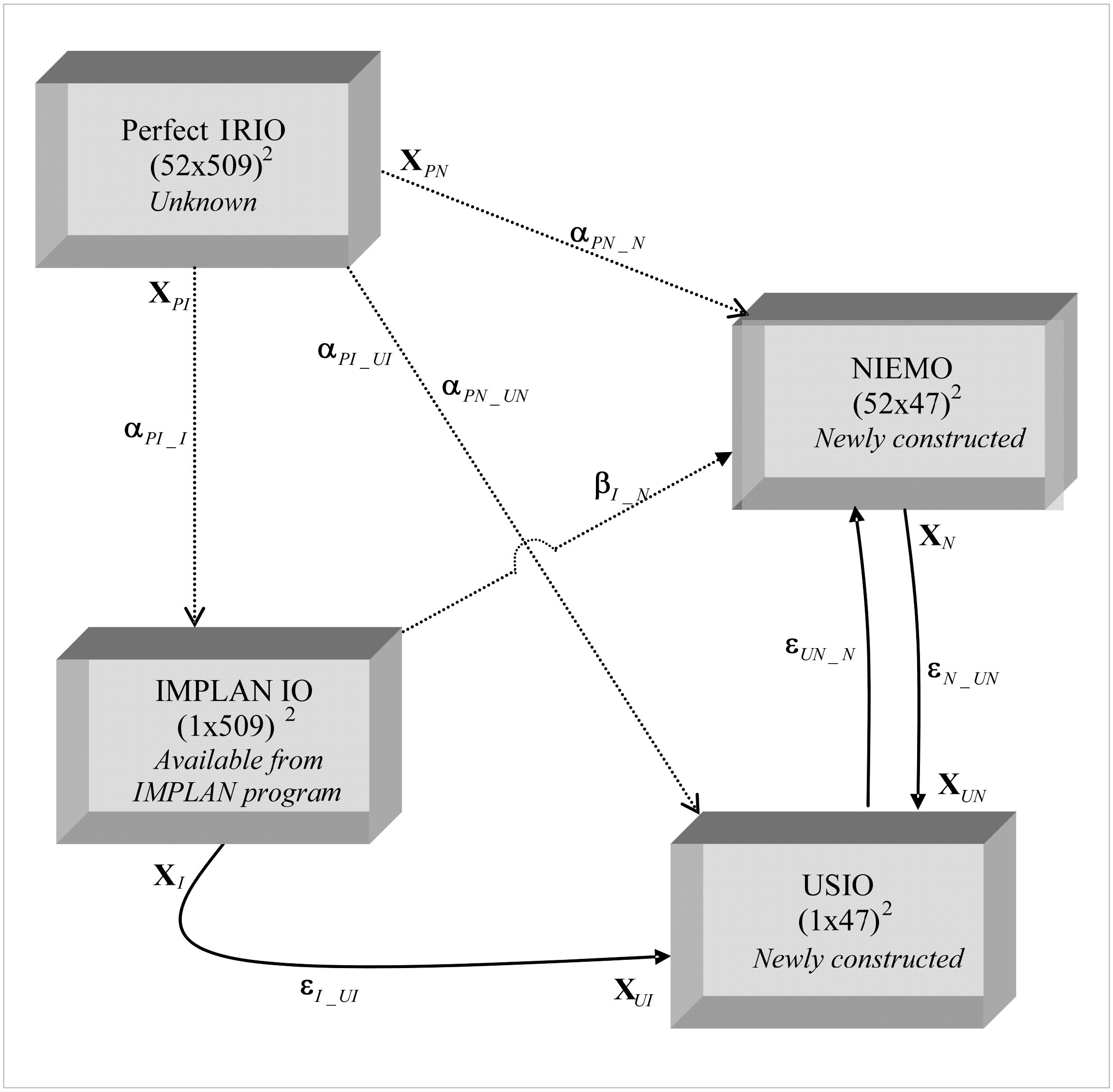
$\mathbf{\Gamma}_m$ is the vector of inputs for IO model m. For the comparison of national IMPLAN IO and USIO; $\mathbf{\Gamma}$ is 509 elements of one dollar if m is the IMPLAN IO model and dollar values aggregated to 47 USC sectors shown in the second column of Table 2 if m is the USIO model. For the comparison of USIO and NIEMO, $\mathbf{\Gamma}$ is \$51 for each element if m is the USIO model, and \$1 for each element if m is NIEMO, as shown in the second column of Table 3.

Based on the three models introduced in equation (6), Figure 2 elaborates the approach used in this study, based on Figure 1. One contrast is IMPLAN IO vs. USIO for comparison of sectoral aggregation errors; another is NIEMO vs. USIO for compariosn of spatial aggregation errors. Figure 2 highlights errors that are plausibly encountered as I attempt to estimate any one of the models from the others. Except for the disaggregation error $\boldsymbol{\varepsilon}_{UN\_N}$ and the unknown mixed error $\boldsymbol{\beta}_{I\_N}$, all other errors shown in the Figure 2 are aggregation errors.

[7] These errors might arise from rounding decimal points when creating the USIO_U matrix and combining the data.

To understand the nature of the various errors, however, it should be pointed out that some of them, the $\boldsymbol{\varepsilon}$s, can be estimated from the models depicted. The others ($\boldsymbol{\alpha}$s and $\boldsymbol{\beta}$) are conceptual. The following shows that the four unknown errors can be inferred from the estimated errors, but the notations necessary to calculate the errors will be explained first.

Figure 2. Errors resulting from various types of IO models



Notations of Total Outputs

$\mathbf{X}_{Pi}$: Total output vector aggregated from 509 IMPLAN sectors and 52 regions via the *unknown* perfect IRIO to 47 USC sectors and the whole U.S., where $i = N$ (for the case of the same total input as NIEMO) or I (for the case of the same total input as IMPLAN IO).

$\mathbf{X}_N$: Total output vector, spatially aggregated from 47 USC sectors and 52 regions via NIEMO.

$\mathbf{X}_I$: Total output vector, sectorally aggregated from 509 IMPLAN sectors and the U.S., via the available IMPLAN IO for the U.S., from the IMPLAN program

$\mathbf{X}_{Ui}$: Total output vector of 47 USC sectors and the U.S. via USIO, where $i = N$ (for the case of the same total input as NIEMO) or I (for the case of the same total input as IMPLAN IO).

Assumptions

$\boldsymbol{\alpha}_{PI\_I}$ = $\boldsymbol{\varepsilon}_{N\_UN}$: Unknown vector of spatial aggregation errors are the same as calculated vector of spatial aggregation errors. (7.1)

$\boldsymbol{\alpha}_{PN\_N}$ = $\boldsymbol{\varepsilon}_{I\_UI}$: Unknown vector of sectoral aggregation errors are the same as calculated vector of sectoral aggregation errors. (7.2)

Based on the total output notations and the basic assumptions, a discussion of the errors can be constructed, as in equations (8.).

Unknown Aggregation Errors

$\alpha^j_{PI\_I} = \dfrac{x^j_{PI} - x^j_I}{x^j_{PI}}$: Unknown element j in vector $\boldsymbol{\alpha}_{PI\_I}$ of spatial aggregation errors of IMPLAN IO stemming from the perfect IRIO. (8.1)

$\alpha^j_{PN\_N} = \dfrac{x^j_{PN} - x^j_N}{x^j_{PN}}$: Unknown element j in vector $\boldsymbol{\alpha}_{PN\_N}$ of sectoral aggregation errors of NIEMO stemming from the perfect IRIO. (8.2)

$\alpha_{Pi\_Ui}^{j} = \dfrac{x_{Pi}^{j} - x_{Ui}^{j}}{x_{Pi}^{j}}$: Unknown element j in vector $\boldsymbol{\alpha}_{Pi\_Ui}$ of sectoral and spatial aggregation errors of USIO stemming from the perfect IRIO, where $i = N$ (for the case of the same total input as NIEMO) or I (for the case of the same total input as IMPLAN IO). (8.3)

Unknown Mixed Errors

$\boldsymbol{\beta}_{I\_N}$: Unknown vector including errors by sectoral aggregation and information by disaggregation spatially from IMPLAN IO to NIEMO.

Calculated Aggregation Errors

$\varepsilon_{N\_UN}^{j} = \dfrac{x_{N}^{j} - x_{UN}^{j}}{x_{N}^{j}}$: Calculated element j in vector $\boldsymbol{\varepsilon}_{N\_UN}$ of spatial aggregation errors of USIO stemming from NIEMO. (9.1)

$\varepsilon_{I\_UI}^{j} = \dfrac{x_{I}^{j} - x_{UI}^{j}}{x_{I}^{j}}$: Calculated element j in vector $\boldsymbol{\varepsilon}_{I\_UI}$ of sectoral aggregation errors of USIO stemming from the IMPLAN IO. (9.2)

Calculated Disaggregation Errors

$\varepsilon_{UN\_N}^{j} = \dfrac{x_{UN}^{j} - x_{N}^{j}}{x_{UN}^{j}}$: Calculated element j in vector $\boldsymbol{\varepsilon}_{UN\_N}$ of spatial disaggregation errors of NIEMO stemming from USIO. (10)

First, based on the assumptions of equations (8.), the total output vector of the perfect IRIO, $\mathbf{X}_{Pi}$ (where $i = N$ or I), can be estimated as,

$$\alpha_{PI\_I}^{j} = \frac{x_{PI}^{j} - x_{I}^{j}}{x_{PI}^{j}} \qquad (11.1)$$

$$=1-\frac{x_I^j}{x_{PI}^j}, \tag{11.2}$$

and, by assumption of equation (7.1),

$$\alpha_{PI\_I}^j = \varepsilon_{N\_UN}^j = 1-\frac{x_{UN}^j}{x_N^j}. \tag{12}$$

Hence,

$$x_{PI}^j = \frac{x_N^j x_I^j}{x_{UN}^j}. \tag{13}$$

Similarly, based on the assumption of equation (7.2),

$$x_{PN}^j = \frac{x_N^j x_I^j}{x_{UI}^j}. \tag{14}$$

Further, vector $\boldsymbol{\varepsilon}$ s are calculated from the relation of the models, two estimated error vectors $\hat{\boldsymbol{\alpha}}_{PI\_I}$ and $\hat{\boldsymbol{\alpha}}_{PN\_N}$ can be obtained as,

$$\hat{\boldsymbol{\alpha}}_{PI\_I} = \boldsymbol{\varepsilon}_{N\_UN} \text{, and} \tag{15.1}$$

$$\hat{\boldsymbol{\alpha}}_{PN\_N} = \boldsymbol{\varepsilon}_{I\_UI}. \tag{15.2}$$

Based on the defined vectors from equations (15.), the rest are estimated with equations (13) and (14) showing that $\hat{\boldsymbol{\alpha}}_{PN\_UN} = \hat{\boldsymbol{\alpha}}_{PI\_UI} \equiv \hat{\boldsymbol{\alpha}}_{P\_U}$, as shown below.

$$\hat{\alpha}_{PN\_UN}^j = 1-\frac{x_{UN}^j}{x_{PN}^j} \tag{16.1}$$

$$= \ 1-\frac{x_{UI}^{j}x_{UN}^{j}}{x_{N}^{j}x_{I}^{j}} \tag{16.2}$$

$$= \ 1-\frac{x_{UN}^{j}x_{UI}^{j}}{x_{I}^{j}x_{N}^{j}} \tag{16.3}$$

$$= \ \hat{\alpha}_{PI\_UI}^{j} \tag{16.4}$$

$$\equiv \ \hat{\alpha}_{P\_U}^{j} \tag{16.5}$$

Although the vector $\boldsymbol{\beta}_{I\_N}$ can be defined as,

$$\beta_{I\_N}^{j} = \frac{x_I^j - x_N^j}{x_I^j} = 1-\frac{x_N^j}{x_I^j}, \tag{17}$$

if x_I^j and x_N^j are total outputs for element j with the same inputs from national IMPLAN IO model and NIEMO, respectively, equation (17) would not be available because different inputs for the two models are used in this study.

Nevertheless, because the USIO model is based on the same data as both IO models, $\frac{x_N^j / x_{UN}^j}{x_I^j / x_{UI}^j}$ can be substituted for the $\frac{x_N^j}{x_I^j}$ shown in equation (17), where x_N^j and x_I^j are adjusted to the same scales of values based on USIO total outputs. Therefore,

$$\hat{\beta}_{I\_N}^{j} = \ 1-\frac{x_N^j / x_{UN}^j}{x_I^j / x_{UI}^j} \tag{18.1}$$

$$= \ 1-\frac{(1-\varepsilon_{UN\_N}^{j})}{(1-\varepsilon_{UI\_I}^{j})} \tag{18.2}$$

$$= \ 1-(1-\varepsilon_{UN\_N}^{j})(1-\varepsilon_{I\_UI}^{j}) \tag{18.3}$$

$$= \ \varepsilon_{I\_UI}^{j} + \varepsilon_{UN\_N}^{j} - \varepsilon_{I\_UI}^{j} * \varepsilon_{UN\_N}^{j} \tag{18.4}$$

$$= \ \varepsilon_{I\_UI}^{j} + \varepsilon_{UN\_N}^{j} - \Xi_{I\_N}^{j} \tag{18.5}$$

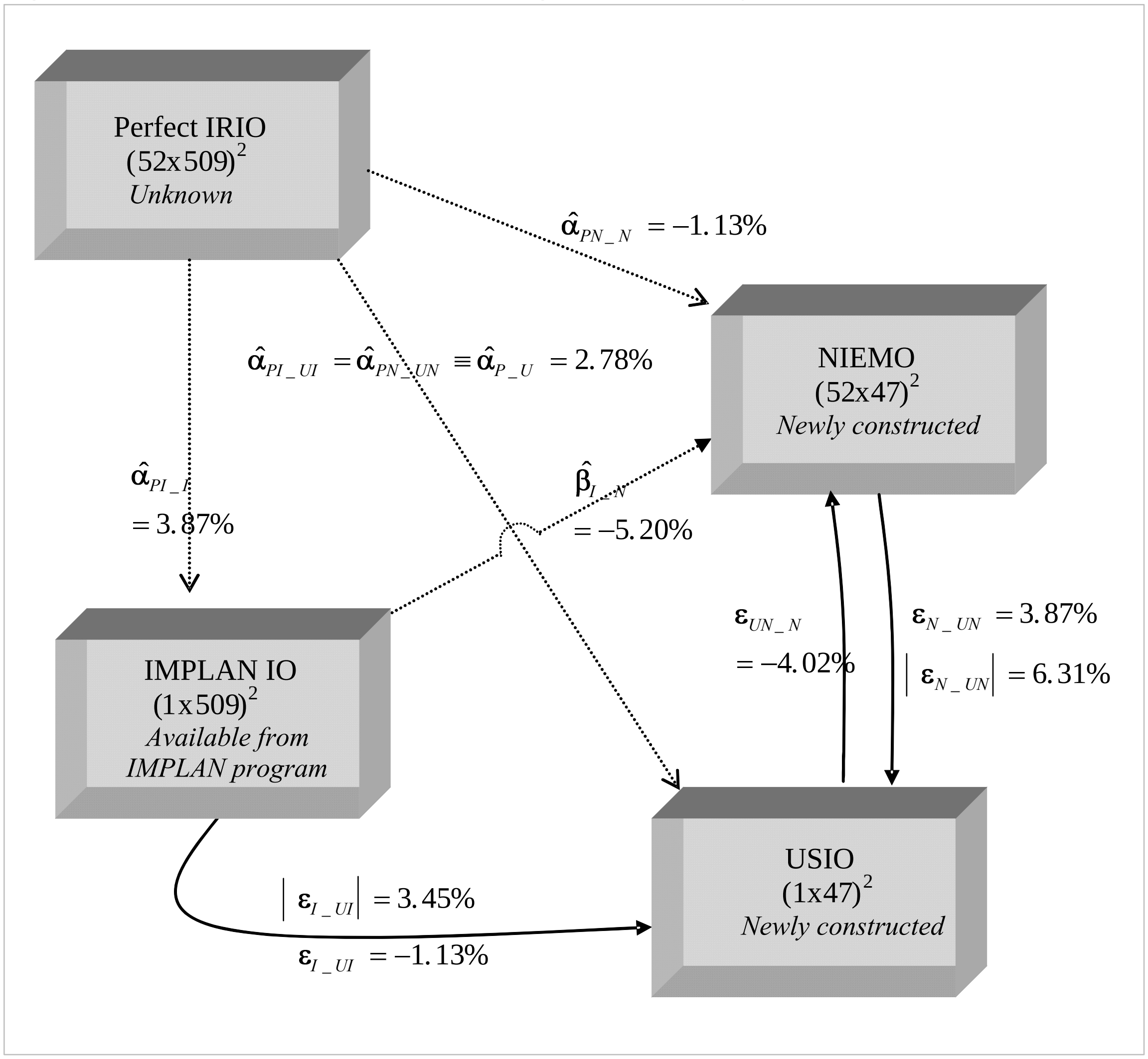
where, $\Xi_{I\_N}^{j} \equiv \varepsilon_{I\_UI}^{j} * \varepsilon_{UN\_N}^{j}$, indicates the degree of j sector accuracy in the model.

IV. Results

Figure 3 shows the overall estimated and calculated errors resulting from various types of IO models matched to the errors suggested in Figure 2. I find that the vector of $\hat{\boldsymbol{\beta}}_{I\_N}$ (column 8 of Table 4) denotes an overall error of 5.2 percent. Yet, this error is difficult to put into perspective because it is not known whether IMPLAN or NIEMO provides the better estimate of the ideal model. My approach suggests an answer in columns 4 and 5 of Table 4, which reveals that spatial aggregation introduces an overall larger error (=3.87 percent) than sectoral aggregation (=-1.13 percent). Tables 2 and 3 include the corresponding absolute values of errors which have the same relationship, respectively 3.45 as sectoral aggregation errors and 6.31 percent as spatial aggregation errors. This indicates spatial aggregation might produce more severe aggregation problems, that is, the cost of choosing the IMPLAN national model is higher than that to use NIEMO if the perfect IRIS is accurate.

Also, based on these aggregation errors, the perfect IRIO vector $\mathbf{X}_{PI}$ and $\mathbf{X}_{PN}$ can be estimated easily, as shown in column 2 and column 3 of Table 4. The estimates for total sum value of the vector $\mathbf{X}_{PI}$ (= $\sum_j x_{PI}^j$), and that of $\mathbf{X}_{PN}$ (= $\sum_j x_{PN}^j$) are \$958 and \$4,469 respectively, showing 3.87 percent for the overall aggregation error between IRIO and IMPLAN IO and –1.13 percent for that between IRIO and NIEMO. Therefore, NIEMO is better at reflecting the perfect IRIO than the IMPLAN IO, with smaller exaggeration, by 1.13 percent (column 2 of Table 4). The aggregation errors from IRIO to USIO (= $\hat{\boldsymbol{\alpha}}_{P\_U}$) are close to the sum of $\hat{\boldsymbol{\alpha}}_{PN\_N}$ and $\boldsymbol{\varepsilon}_{N\_UN}$ or the sum of $\hat{\boldsymbol{\alpha}}_{PI\_I}$ and $\boldsymbol{\varepsilon}_{I\_UI}$, or $\hat{\boldsymbol{\alpha}}_{PN\_N}$ + $\hat{\boldsymbol{\alpha}}_{PI\_I}$, because it is derived as their products. The following discussion elaborates the approach and findings further.

Figure 3. Overall estimated errors resulting from various types of IO models



Note: The hat on the vector $\boldsymbol{\alpha}$ s and $\boldsymbol{\beta}$ indicates the estimates of errors.

NIEMO Benefits from Spatial Information: $\boldsymbol{\varepsilon}_{UN\_N}$

Figure 2 highlighted the fact that disaggregation errors $\boldsymbol{\varepsilon}_{UN\_N}$ add spatial information to the sectorally aggregated IO model (USIO), making the simple one-region USIO model spatially detailed. Because the simple USIO model has no detailed information for states or subregions, the overall estimates are smaller than the spatially distributed NIEMO, although the simple USIO model may have the possibility of under- (36 sectors) or over- (11 sectors) estimating the various NIEMO sectors.

Table 2. Sectoral aggregation errors between 47 USIO and 509 IMPLAN IO (units of $\mathbf{X}$ s: $)

		Total Impacts		Sectoral Aggregation Errors			
USC Sectors	Direct Impacts*	$\mathbf{X}_I$ **	$\mathbf{X}_{UI}$	$\mathbf{X}_I - \mathbf{X}_{UI}$	$\boldsymbol{\varepsilon}_{I\_UI}$	$\lvert\mathbf{X}_I - \mathbf{X}_{UI}\rvert$	$\lvert\boldsymbol{\varepsilon}_{I\_UI}\rvert$
USC 1	8.25	12.25	14.94	-2.69	-21.98%	2.69	21.98%
USC 2	10.50	18.67	16.70	1.97	10.57%	1.97	10.57%
USC 3	2.25	4.80	5.61	-0.81	-16.82%	0.81	16.82%
USC 4	10.33	11.66	11.68	-0.02	-0.16%	0.02	0.16%
USC 5	22.08	29.87	31.38	-1.51	-5.06%	1.51	5.06%
USC 6	4.00	6.43	6.47	-0.04	-0.59%	0.04	0.59%
USC 7	3.00	3.22	3.11	0.12	3.60%	0.12	3.60%
USC 8	3.33	5.56	5.39	0.17	3.03%	0.17	3.03%
USC 9	3.33	5.45	5.24	0.21	3.80%	0.21	3.80%
USC 10	6.50	25.08	24.59	0.49	1.95%	0.49	1.95%
USC 11	6.33	11.72	11.25	0.47	4.00%	0.47	4.00%
USC 12	2.00	2.47	2.57	-0.10	-4.02%	0.10	4.02%
USC 13	2.50	3.97	3.67	0.30	7.57%	0.30	7.57%
USC 14	11.33	19.25	18.91	0.35	1.81%	0.35	1.81%
USC 15	14.50	26.52	26.53	0.00	-0.01%	0.00	0.01%
USC 16	13.00	22.63	23.27	-0.64	-2.84%	0.64	2.84%
USC 17	9.83	21.77	20.91	0.86	3.96%	0.86	3.96%
USC 18	7.00	13.30	13.52	-0.22	-1.69%	0.22	1.69%
USC 19	22.58	31.21	31.42	-0.21	-0.67%	0.21	0.67%
USC 20	21.17	25.83	26.36	-0.53	-2.05%	0.53	2.05%
USC 21	19.50	37.14	36.02	1.12	3.00%	1.12	3.00%
USC 22	25.75	38.86	39.68	-0.83	-2.13%	0.83	2.13%
USC 23	50.83	62.09	62.37	-0.28	-0.45%	0.28	0.45%
USC 24	33.58	46.97	49.12	-2.15	-4.58%	2.15	4.58%
USC 25	10.83	16.22	18.38	-2.16	-13.30%	2.16	13.30%
USC 26	5.00	5.83	5.83	0.00	0.02%	0.00	0.02%
USC 27	12.50	13.50	13.66	-0.16	-1.18%	0.16	1.18%
USC 28	12.00	12.80	12.75	0.05	0.42%	0.05	0.42%
USC 29	20.17	23.34	23.33	0.00	0.02%	0.00	0.02%
USC 30	3.00	13.79	12.95	0.84	6.09%	0.84	6.09%
USC 31	13.00	16.45	16.14	0.31	1.87%	0.31	1.87%
USC 32	1.00	42.77	42.62	0.15	0.35%	0.15	0.35%
USC 33	7.00	28.49	27.03	1.46	5.13%	1.46	5.13%
USC 34	3.00	9.53	9.39	0.14	1.42%	0.14	1.42%
USC 35	12.00	16.41	17.03	-0.62	-3.77%	0.62	3.77%
USC 36	6.00	19.31	20.06	-0.75	-3.87%	0.75	3.87%
USC 37	6.00	25.48	27.29	-1.81	-7.11%	1.81	7.11%
USC 38	7.00	33.48	33.96	-0.48	-1.42%	0.48	1.42%
USC 39	14.00	41.73	41.46	0.27	0.65%	0.27	0.65%
USC 40	1.00	14.39	13.91	0.48	3.36%	0.48	3.36%
USC 41	9.00	22.25	22.84	-0.58	-2.62%	0.58	2.62%
USC 42	2.00	2.24	2.28	-0.05	-2.05%	0.05	2.05%
USC 43	8.00	9.56	9.25	0.31	3.27%	0.31	3.27%
USC 44	8.00	10.74	10.13	0.61	5.67%	0.61	5.67%
USC 45	3.00	7.11	7.38	-0.27	-3.83%	0.27	3.83%
USC 46	9.00	8.94	12.60	-3.66	-40.98%	3.66	40.98%
USC 47	18.00	39.99	40.52	-0.53	-1.32%	0.53	1.32%
TOTAL	504.00	921	931	-10.42	-1.13%	31.78	3.45%

* 509 IMPLAN IO model actually uses $1 inputs for 504 sectors, excluding 5 sectors, or 501-503 and 507-508, due to the nonexistence of coefficients for those sectors

** Results of IMPLAN IO are aggregated to 47 USC sectors, based on the bridge of IMPLAN and USC after being calculated from 509-sector IMPLAN IO models

When total outputs via NIEMO and via USIO are compared in Table 3 and 4, for instance, USIO underestimates \$174.71 overall, showing 3.87 percent as total aggregation error from NIEMO and -4.02 percent as total disaggregation error for NIEMO, based on the same inputs. Therefore, the total disaggregation error of $\boldsymbol{\varepsilon}_{UN\_N}$ indicates the overall percentage of spatial information required to adjust a simplified one-region model and also that NIEMO adds benefits by 4.02 percent from disaggregating spatially.

NIEMO costs: $\hat{\boldsymbol{\beta}}_{I\_N}$

The estimated vector $\hat{\boldsymbol{\beta}}_{I\_N}$ (47x1) includes mixed information: benefits from information gained from disaggregating spatially ($\boldsymbol{\varepsilon}_{UN\_N}$) vs. costs from sectoral aggregations ($\boldsymbol{\varepsilon}_{I\_UI}$). Via the definitions and calculations of various errors ($\boldsymbol{\varepsilon}$) and IO models shown in Figure 2, vector $\hat{\boldsymbol{\beta}}_{I\_N}$ includes the sum of two errors ($\boldsymbol{\varepsilon}_{I\_UI}$ and $\boldsymbol{\varepsilon}_{UN\_N}$) adjusted by their product. Although there might be spatial aggregation errors in the IMPLAN U.S. national IO model, compared to the perfect IRIO-type model with 52 regions, IMPLAN has been widely used for more than twenty years to in various U.S. impact studies. The vector $\hat{\boldsymbol{\beta}}_{I\_N}$ indicates, in this sense, how close the results of NIEMO and IMPLAN U.S. IO are, even though NIEMO, derived from state-level coefficients of IMPLAN IO models, simultaneously includes both information to improve model accuracy and errors to aggravate this. This means that if the overall difference of $\hat{\boldsymbol{\beta}}_{I\_N}$, is large enough, then a spatially disaggregated model such as NIEMO might be an unacceptable model.

A good indicator to reflect the credibility of NIEMO, therefore, can be the absolute values of total percentage errors shown in Table 4. Because the results reveal that the absolute overall difference of $\hat{\boldsymbol{\beta}}_{I\_N}$ is 5.20 percent, I can obtain detailed information for each state by USC sector, with a loss vis-à-vis the widely accepted IMPLAN model of 5.2 percent, denoting the costs for adopting this MRIO-type model.

Table 3. Spatial aggregation errors between NIEMO and USIO (units of $\mathbf{X}$ s: $)

USC Sectors	Direct Impacts[*]	Total Impacts		Spatial Aggregation Errors			
		$\mathbf{X}_N$ **	$\mathbf{X}_{UN}$	$\mathbf{X}_N - \mathbf{X}_{UN}$	$\boldsymbol{\varepsilon}_{N\_UN}$	$\lvert\mathbf{X}_N - \mathbf{X}_{UN}\rvert$	$\lvert\boldsymbol{\varepsilon}_{N\_UN}\rvert$
USC 1	51.00	84.70	86.40	-1.70	-2.00%	1.70	2.00%
USC 2	51.00	83.45	85.26	-1.80	-2.16%	1.80	2.16%
USC 3	51.00	70.92	70.77	0.15	0.21%	0.15	0.21%
USC 4	51.00	59.19	58.95	0.25	0.41%	0.25	0.41%
USC 5	51.00	99.87	100.57	-0.70	-0.70%	0.70	0.70%
USC 6	51.00	65.84	63.29	2.55	3.87%	2.55	3.87%
USC 7	51.00	52.34	52.50	-0.15	-0.29%	0.15	0.29%
USC 8	51.00	63.14	60.35	2.78	4.41%	2.78	4.41%
USC 9	51.00	62.26	62.07	0.19	0.31%	0.19	0.31%
USC 10	51.00	180.71	161.33	19.38	10.73%	19.38	10.73%
USC 11	51.00	87.16	76.38	10.78	12.37%	10.78	12.37%
USC 12	51.00	57.97	57.82	0.15	0.25%	0.15	0.25%
USC 13	51.00	61.12	59.19	1.93	3.17%	1.93	3.17%
USC 14	51.00	85.84	82.34	3.50	4.08%	3.50	4.08%
USC 15	51.00	103.78	99.83	3.95	3.81%	3.95	3.81%
USC 16	51.00	99.09	93.54	5.54	5.60%	5.54	5.60%
USC 17	51.00	104.22	102.30	1.92	1.84%	1.92	1.84%
USC 18	51.00	67.47	85.51	-18.05	-26.75%	18.05	26.75%
USC 19	51.00	72.29	78.44	-6.15	-8.51%	6.15	8.51%
USC 20	51.00	72.22	70.17	2.05	2.84%	2.05	2.84%
USC 21	51.00	111.02	93.95	17.07	15.38%	17.07	15.38%
USC 22	51.00	98.22	95.14	3.08	3.14%	3.08	3.14%
USC 23	51.00	93.71	89.73	3.97	4.24%	3.97	4.24%
USC 24	51.00	118.55	101.64	16.91	14.26%	16.91	14.26%
USC 25	51.00	78.71	81.59	-2.87	-3.65%	2.87	3.65%
USC 26	51.00	56.00	56.53	-0.53	-0.94%	0.53	0.94%
USC 27	51.00	56.99	56.56	0.43	0.76%	0.43	0.76%
USC 28	51.00	53.41	54.27	-0.86	-1.61%	0.86	1.61%
USC 29	51.00	79.69	61.69	17.99	22.58%	17.99	22.58%
USC 30	51.00	107.31	97.23	10.09	9.40%	10.09	9.40%
USC 31	51.00	69.69	67.98	1.72	2.47%	1.72	2.47%
USC 32	51.00	227.75	218.81	8.95	3.93%	8.95	3.93%
USC 33	51.00	155.65	144.63	11.02	7.08%	11.02	7.08%
USC 34	51.00	84.00	82.15	1.85	2.20%	1.85	2.20%
USC 35	51.00	76.02	74.29	1.72	2.27%	1.72	2.27%
USC 36	51.00	111.89	122.17	-10.28	-9.19%	10.28	9.19%
USC 37	51.00	165.11	159.25	5.86	3.55%	5.86	3.55%
USC 38	51.00	213.64	198.72	14.92	6.98%	14.92	6.98%
USC 39	51.00	225.20	184.29	40.91	18.17%	40.91	18.17%
USC 40	51.00	110.82	109.70	1.12	1.01%	1.12	1.01%
USC 41	51.00	121.02	117.62	3.40	2.81%	3.40	2.81%
USC 42	51.00	53.92	52.56	1.36	2.52%	1.36	2.52%
USC 43	51.00	62.94	60.64	2.29	3.65%	2.29	3.65%
USC 44	51.00	63.09	62.77	0.32	0.51%	0.32	0.51%
USC 45	51.00	72.63	71.27	1.36	1.87%	1.36	1.87%
USC 46	51.00	56.08	68.26	-12.18	-21.72%	12.18	21.72%
USC 47	51.00	162.50	153.99	8.51	5.24%	8.51	5.24%
TOTAL	2397.00	4,519	4,344	174.71	3.87%	285.25	6.31%

*NIEMO actually uses $1 inputs for 51 states and 47 sectors, then aggregates the inputs by USC sector, with $51 excluding rest of world.

**Results of NIEMO aggregate those of 51 states by USC sector.

Furthermore, if NIEMO is true, then each sectoral error in vector $\hat{\boldsymbol{\beta}}_{I\_N}$ denotes that the corresponding result for IMPLAN national IO can be adjusted by NIEMO by the amount of $\varepsilon^{j}_{UN\_N}$ - $\varepsilon^{j}_{I\_UI} * \varepsilon^{j}_{UN\_N}$, which is the spatial dissaggregation of sector j and the model accuracy term.

Model Accuracy: $\boldsymbol{\Xi}_{I\_N}$

Finally, model accuracy can also be represented by the product of two errors, or $\varepsilon^{j}_{I\_UI} * \varepsilon^{j}_{UN\_N}$ defined as $\Xi^{j}_{I\_N}$. In the previous discussion, I noted that $\hat{\boldsymbol{\beta}}_{I\_N}$ might be equal to the sum of $\boldsymbol{\varepsilon}_{I\_UI}$ and $\boldsymbol{\varepsilon}_{UN\_N}$. However, if both errors are exceptionally large, then the sector or model itself will be far different from the normal sums due to the error product. Therefore, the product can be used as another indicator to monitor model accuracy, that is, whether the case of mixed aggregation contains severely distorted sectors (or their aggregate) or not. In the calculated results, there are only minor distortions because all $\Xi^{j}_{I\_N}$ are smaller than 1 percent in absolute value, except the USC 46 (Public Administration) Sector.

V. Conclusions

This study, based on the newly constructed NIEMO and USIO models, reports on tests of the accuracy of the approach. I use as a benchmark the widely used IMPLAN IO model. While the IMPLAN IO models have sectorally detailed information but lack spatially disaggregated detail, NIEMO develops spatially detailed information at the cost of aggregated industrial sectors. As a common reference model, I constructed USIO, which has aggregated sectors corresponding to those used for NIEMO but only one aggregate national area as does the IMPLAN national IO model. This was constructed to compare spatial and sectoral aggregation errors. Also, with respect to spatial disaggregation errors, I tested NIEMO's sectoral aggregation cost in terms of model accuracy incurred by switching from the IMPLAN IO model to NIEMO.

I have shown that a multiregional input-output model for the U.S. containing approximately six-million multipliers can be constructed at low cost given the fact that IMPLAN's input-output matrices are plausible. With respect to the estimation of overall model accuracy, I found only relatively small errors when comparing the aggregates of all sectors and also only minor errors on an individual sector-by-sector comparison basis.

I have also demonstrated that the sectoral aggregation required to go from IMPLAN to NIEMO imparts only minor errors. I conclude that, all things considered, it is more useful to construct an MRIO-type model instead of a general one-region IO model, especially for a large region that includes many economically diverse subregions.

Nevertheless, NIEMO requires further work. The next steps in the research program involve relaxing the assumption of no intestate trade in services and, instead, estimating interstate trade flows for the 18 service sectors of the USC-sector system. This may address some of the underestimated impacts and spatial aggregation errors vis a vis the perfect IRIO model. In this sense, the recent suggestions by Park (2006b) will help to estimate origin-destination matrices for the service industries. This requires using the state-level domestic imports and exports vector available for each state and applying Geographically Weighted Regressions (GWR) along the lines suggested by LeSage (1999).

Table 4. Estimates of $\mathbf{X}_{PI}$ and $\mathbf{X}_{PN}$, and various other errors (units of $\mathbf{X}$s: $)

USC Sectors	$\mathbf{X}_{PI}$	$\mathbf{X}_{PN}$	$\hat{\boldsymbol{\alpha}}_{PN\_N}$	$\hat{\boldsymbol{\alpha}}_{PI\_I}$	$\hat{\boldsymbol{\alpha}}_{P\_U}$	$\boldsymbol{\varepsilon}_{UN\_N}$	$\hat{\boldsymbol{\beta}}_{I\_N}$	$\boldsymbol{\Xi}_{I\_N}$
USC 1	12.01	69.44	-21.98%	-2.00%	-24.43%	1.96%	-19.59%	-0.43%
USC 2	18.27	93.32	10.57%	-2.16%	8.64%	2.11%	12.46%	0.22%
USC 3	4.81	60.71	-16.82%	0.21%	-16.58%	-0.21%	-17.07%	0.04%
USC 4	11.71	59.10	-0.16%	0.41%	0.25%	-0.42%	-0.58%	0.00%
USC 5	29.66	95.06	-5.06%	-0.70%	-5.80%	0.70%	-4.33%	-0.04%
USC 6	6.69	65.45	-0.59%	3.87%	3.30%	-4.02%	-4.64%	0.02%
USC 7	3.21	54.30	3.60%	-0.29%	3.32%	0.29%	3.88%	0.01%
USC 8	5.82	65.11	3.03%	4.41%	7.31%	-4.61%	-1.44%	-0.14%
USC 9	5.47	64.72	3.80%	0.31%	4.10%	-0.31%	3.49%	-0.01%
USC 10	28.10	184.30	1.95%	10.73%	12.46%	-12.01%	-9.83%	-0.23%
USC 11	13.38	90.79	4.00%	12.37%	15.88%	-14.11%	-9.54%	-0.56%
USC 12	2.47	55.73	-4.02%	0.25%	-3.76%	-0.25%	-4.28%	0.01%
USC 13	4.10	66.12	7.57%	3.17%	10.49%	-3.27%	4.54%	-0.25%
USC 14	20.07	87.43	1.81%	4.08%	5.82%	-4.25%	-2.36%	-0.08%
USC 15	27.57	103.77	-0.01%	3.81%	3.80%	-3.96%	-3.97%	0.00%
USC 16	23.97	96.35	-2.84%	5.60%	2.92%	-5.93%	-8.93%	0.17%
USC 17	22.18	108.52	3.96%	1.84%	5.73%	-1.87%	2.17%	-0.07%
USC 18	10.49	66.35	-1.69%	-26.75%	-28.89%	21.10%	19.77%	-0.36%
USC 19	28.76	71.80	-0.67%	-8.51%	-9.24%	7.84%	7.22%	-0.05%
USC 20	26.59	70.77	-2.05%	2.84%	0.85%	-2.93%	-5.04%	0.06%
USC 21	43.89	114.46	3.00%	15.38%	17.92%	-18.18%	-14.63%	-0.55%
USC 22	40.11	96.18	-2.13%	3.14%	1.07%	-3.24%	-5.44%	0.07%
USC 23	64.83	93.28	-0.45%	4.24%	3.81%	-4.43%	-4.90%	0.02%
USC 24	54.79	113.36	-4.58%	14.26%	10.34%	-16.64%	-21.98%	0.76%
USC 25	15.65	69.47	-13.30%	-3.65%	-17.44%	3.52%	-9.32%	-0.47%
USC 26	5.77	56.01	0.02%	-0.94%	-0.93%	0.93%	0.95%	0.00%
USC 27	13.60	56.32	-1.18%	0.76%	-0.41%	-0.77%	-1.96%	0.01%
USC 28	12.60	53.64	0.42%	-1.61%	-1.19%	1.58%	1.99%	0.01%
USC 29	30.15	79.70	0.02%	22.58%	22.59%	-29.17%	-29.14%	-0.01%
USC 30	15.22	114.27	6.09%	9.40%	14.91%	-10.37%	-3.66%	-0.63%
USC 31	16.87	71.02	1.87%	2.47%	4.29%	-2.53%	-0.61%	-0.05%
USC 32	44.52	228.55	0.35%	3.93%	4.26%	-4.09%	-3.73%	-0.01%
USC 33	30.66	164.07	5.13%	7.08%	11.85%	-7.62%	-2.09%	-0.39%
USC 34	9.74	85.21	1.42%	2.20%	3.59%	-2.25%	-0.79%	-0.03%
USC 35	16.79	73.26	-3.77%	2.27%	-1.42%	-2.32%	-6.17%	0.09%
USC 36	17.69	107.72	-3.87%	-9.19%	-13.42%	8.42%	4.87%	-0.33%
USC 37	26.42	154.16	-7.11%	3.55%	-3.30%	-3.68%	-11.05%	0.26%
USC 38	36.00	210.64	-1.42%	6.98%	5.66%	-7.51%	-9.04%	0.11%
USC 39	50.99	226.66	0.65%	18.17%	18.69%	-22.20%	-21.41%	-0.14%
USC 40	14.54	114.67	3.36%	1.01%	4.33%	-1.02%	2.37%	-0.03%
USC 41	22.90	117.92	-2.62%	2.81%	0.26%	-2.89%	-5.59%	0.08%
USC 42	2.30	52.84	-2.05%	2.52%	0.53%	-2.59%	-4.69%	0.05%
USC 43	9.92	65.07	3.27%	3.65%	6.80%	-3.78%	-0.39%	-0.12%
USC 44	10.79	66.88	5.67%	0.51%	6.16%	-0.51%	5.19%	-0.03%
USC 45	7.24	69.95	-3.83%	1.87%	-1.89%	-1.90%	-5.80%	0.07%
USC 46	7.34	39.78	-40.98%	-21.72%	-71.59%	17.84%	-15.83%	-7.31%
USC 47	42.20	160.38	-1.32%	5.24%	3.99%	-5.53%	-6.92%	0.07%
TOTAL	958	4,469	-1.13%	3.87%	2.78%	-4.02%	-5.20%	0.05%

Note: The hat on the vector $\boldsymbol{\alpha}$s and $\boldsymbol{\beta}$ indicates the estimates of errors.

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Appendix 1. Comparison of value added between IMPLAN and BEA data sets

USC	Total Value Added of U.S. (TVA_US)			Total Value Added of Sum of States (TVA_SS)		
Sector	IMPLAN_US	BEA_US	Diff_US	IMPLAN_SS	BEA_SS	Diff_SS
USC 1	32,149	39,482	-22.8%	32,149	42,815	-33.2%
USC 2	53,336	53,831	-0.9%	53,336	53,559	-0.4%
USC 3	6,573	8,622	-31.2%	6,573	8,914	-35.6%
USC 4	31,985	46,471	-45.3%	31,985	47,955	-49.9%
USC 5	69,117	84,445	-22.2%	69,117	80,324	-16.2%
USC 6	22,754	13,358	41.3%	22,754	13,312	41.5%
USC 7	30,137	9,404	68.8%	30,137	5,324	82.3%
USC 8	11,045	12,054	-9.1%	11,045	11,498	-4.1%
USC 9	4,135	4,499	-8.8%	4,135	4,763	-15.2%
USC 10	89,620	117,465	-31.1%	89,620	117,201	-30.8%
USC 11	25,143	25,180	-0.1%	25,143	24,776	1.5%
USC 12	61,902	62,462	-0.9%	61,902	65,433	-5.7%
USC 13	5,985	5,989	-0.1%	5,985	6,028	-0.7%
USC 14	53,351	53,914	-1.1%	53,351	52,013	2.5%
USC 15	69,988	73,021	-4.3%	69,988	72,812	-4.0%
USC 16	35,716	39,441	-10.4%	35,716	38,064	-6.6%
USC 17	45,101	44,412	1.5%	45,101	44,355	1.7%
USC 18	103,614	96,840	6.5%	103,614	96,764	6.6%
USC 19	57,847	43,529	24.8%	57,847	46,887	18.9%
USC 20	44,457	46,798	-5.3%	44,457	47,025	-5.8%
USC 21	32,230	33,906	-5.2%	32,230	34,080	-5.7%
USC 22	81,546	82,094	-0.7%	81,546	81,729	-0.2%
USC 23	131,457	135,809	-3.3%	131,457	136,426	-3.8%
USC 24	294,755	240,998	18.2%	294,755	242,349	17.8%
USC 25	104,416	111,924	-7.2%	104,416	111,733	-7.0%
USC 26	35,470	41,161	-16.0%	35,470	40,676	-14.7%
USC 27	53,785	45,854	14.7%	53,785	45,268	15.8%
USC 28	30,619	30,466	0.5%	30,619	30,473	0.5%
USC 29	48,510	50,953	-5.0%	48,510	50,497	-4.1%
USC 30	165,574	202,286	-22.2%	165,574	202,287	-22.2%
USC 31	425,995	469,535	-10.2%	425,995	469,533	-10.2%
USC 32	587,645	607,078	-3.3%	587,645	607,076	-3.3%
USC 33	225,713	219,602	2.7%	225,713	220,303	2.4%
USC 34	114,420	77,346	32.4%	114,420	76,644	33.0%
USC 35	579,625	691,578	-19.3%	579,625	691,578	-19.3%
USC 36	339,334	339,077	0.1%	339,334	338,699	0.2%
USC 37	774,908	782,627	-1.0%	774,908	782,625	-1.0%
USC 38	1,269,276	1,276,570	-0.6%	1,269,276	1,276,572	-0.6%
USC 39	784,151	698,825	10.9%	784,151	698,821	10.9%
USC 40	153,223	177,636	-15.9%	153,223	177,635	-15.9%
USC 41	315,921	289,419	8.4%	315,921	289,416	8.4%
USC 42	55,924	59,775	-6.9%	55,924	60,234	-7.7%
USC 43	653,680	679,552	-4.0%	653,680	679,093	-3.9%
USC 44	98,774	95,664	3.1%	98,774	95,661	3.2%
USC 45	259,094	265,805	-2.6%	259,094	265,804	-2.6%
USC 46	1,211,813	1,188,505	1.9%	1,211,813	1,188,506	1.9%
USC 47	395,364	282,894	28.4%	395,364	284,607	28.0%
TOTAL	10,077,179	10,058,158	0.2%	10,077,179	10,058,145	0.2%

Note: Diff_US=(IMPLAN_US-BEA_US)/IMPLAN_US and
Diff_SS=(IMPLAN_SS-BEA_SS)/IMPLAN_SS.
Source: BEA GSP data from http://www.bea.doc.gov/bea/regional/gsp and 2001 IMPLAN data.

Appendix 2. Comparison of total output between IMPLAN and BEA data sets

USC Sector	Total Output of U.S. (TO_US)			Total Output of Sum of States (TO_SS)	
	IMPLAN_US	BEA_US	Diff_US	IMPLAN_SS	Diff_SS
USC 1	173,097	195,922	-13.2%	173,097	0.0%
USC 2	118,853	117,442	1.2%	118,853	0.0%
USC 3	44,785	50,631	-13.1%	44,785	0.0%
USC 4	84,932	105,455	-24.2%	84,932	0.0%
USC 5	286,070	309,327	-8.1%	286,070	0.0%
USC 6	61,546	42,350	31.2%	61,546	0.0%
USC 7	52,637	17,100	67.5%	52,637	0.0%
USC 8	19,049	18,364	3.6%	19,049	0.0%
USC 9	9,129	8,813	3.5%	9,129	0.0%
USC 10	371,603	376,181	-1.2%	371,603	0.0%
USC 11	76,034	75,787	0.3%	76,034	0.0%
USC 12	134,457	134,295	0.1%	134,457	0.0%
USC 13	16,209	16,157	0.3%	16,209	0.0%
USC 14	142,133	142,483	-0.2%	142,133	0.0%
USC 15	203,666	200,586	1.5%	203,666	0.0%
USC 16	101,676	103,276	-1.6%	101,676	0.0%
USC 17	142,353	141,106	0.9%	142,353	0.0%
USC 18	203,883	213,869	-4.9%	203,883	0.0%
USC 19	172,998	130,168	24.8%	172,998	0.0%
USC 20	97,801	97,201	0.6%	97,801	0.0%
USC 21	121,498	118,894	2.1%	121,498	0.0%
USC 22	184,519	182,176	1.3%	184,519	0.0%
USC 23	331,350	322,670	2.6%	331,350	0.0%
USC 24	601,195	608,161	-1.2%	601,195	0.0%
USC 25	447,184	441,871	1.2%	447,184	0.0%
USC 26	118,010	118,170	-0.1%	118,010	0.0%
USC 27	114,130	112,783	1.2%	114,130	0.0%
USC 28	73,637	73,417	0.3%	73,637	0.0%
USC 29	110,923	105,886	4.5%	110,923	0.0%
USC 30	296,699	343,430	-15.8%	296,699	0.0%
USC 31	1,013,113	899,778	11.2%	1,013,113	0.0%
USC 32	875,258	851,286	2.7%	875,258	0.0%
USC 33	502,771	469,464	6.6%	502,771	0.0%
USC 34	162,269	101,953	37.2%	162,269	0.0%
USC 35	942,803	1,021,032	-8.3%	942,803	0.0%
USC 36	586,269	710,579	-21.2%	586,269	0.0%
USC 37	1,287,273	1,361,671	-5.8%	1,287,273	0.0%
USC 38	1,681,503	1,775,397	-5.6%	1,681,503	0.0%
USC 39	1,008,256	1,105,607	-9.7%	1,008,256	0.0%
USC 40	210,209	290,414	-38.2%	210,209	0.0%
USC 41	443,881	481,024	-8.4%	443,881	0.0%
USC 42	85,680	107,147	-25.1%	85,680	0.0%
USC 43	1,188,873	1,094,726	7.9%	1,188,873	0.0%
USC 44	154,279	154,140	0.1%	154,279	0.0%
USC 45	498,852	500,925	-0.4%	498,852	0.0%
USC 46	1,288,980	2,019,174	-56.6%	1,288,980	0.0%
USC 47	755,883	534,941	29.2%	755,883	0.0%
TOTAL	17,598,207	18,403,229	-4.6%	17,598,207	0.0%

Note: Diff_US=(IMPLAN_US-BEA_US)/IMPLAN_US and
Diff_SS=(IMPLAN_SS- IMPLAN _US)/IMPLAN_SS.

Source: BEA GSP data from http://www.bea.doc.gov/bea/regional/gsp and 2001 IMPLAN data.

Appendix 3. Definitions of USC Two-Digit sectors

Classification	USC	Description	SCTG	NAICS
Commodity Sectors	USC01	Live animals and live fish & Meat, fish, seafood, and their preparations	(1+5)	
	USC02	Cereal grains & Other agricultural products except for Animal Feed	(2+3)	
	USC03	Animal feed and products of animal origin, n.e.c.	4	
	USC04	Milled grain products and preparations, and bakery products	6	
	USC05	Other prepared foodstuffs and fats and oils	7	
	USC06	Alcoholic beverages	8	
	USC07	Tobacco products	9	
	USC08	Nonmetallic minerals (Monumental or building stone, Natural sands, Gravel and crushed stone, n.e.c.)	(10~13)	
	USC09	Metallic ores and concentrates	14	
	USC10	Coal and petroleum products (Coal and Fuel oils, n.e.c.)	(15~19)	
	USC11	Basic chemicals	20	
	USC12	Pharmaceutical products	21	
	USC13	Fertilizers	22	
	USC14	Chemical products and preparations, n.e.c.	23	
	USC15	Plastics and rubber	24	
	USC16	Logs and other wood in the rough & Wood products	(25+26)	
	USC17	Pulp, newsprint, paper, and paperboard & Paper or paperboard articles	(27+28)	
	USC18	Printed products	29	
	USC19	Textiles, leather, and articles of textiles or leather	30	
	USC20	Nonmetallic mineral products	31	
	USC21	Base metal in primary or semi-finished forms and in finished basic shapes	32	
	USC22	Articles of base metal	33	
	USC23	Machinery	34	
	USC24	Electronic and other electrical equipment and components, and office equipment	35	
	USC25	Motorized and other vehicles (including parts)	36	
	USC26	Transportation equipment, n.e.c.	37	
	USC27	Precision instruments and apparatus	38	
	USC28	Furniture, mattresses and mattress supports, lamps, lighting fittings, and illuminated signs	39	
	USC29	Miscellaneous manufactured products, Scrap, Mixed freight, and Commodity unknown	(40~99)	
Non-Commodity (Service) Sectors	USC30	Utility		22
	USC31	Construction		23
	USC32	Wholesale Trade		42
	USC33	Transportation		48
	USC34	Postal and Warehousing		49
	USC35	Retail Trade		(44+45)
	USC36	Broadcasting and information services		(515~519)
	USC37	Finance and Insurance		52
	USC38	Real estate and rental and leasing		53
	USC39	Professional, Scientific, and Technical services		54
	USC40	Management of companies and enterprises		55
	USC41	Administrative support and waste management		56
	USC42	Education Services		61
	USC43	Health Care and Social Assistances		62
	USC44	Arts, Entertainment, and Recreation		71
	USC45	Accommodation and Food services		72
	USC46	Public administration		92
	USC47	Other services except public administration		81

Source: Park et al. (2007)