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Planning the trajectories of land and development rights rents via discrete programming models

P Gordon
School of Urban and Regional Planning and Department of Economics, University of Southern California, Los Angeles, CA 90089-0042, USA
J E Moore II
School of Urban and Regional Planning and Department of Civil Engineering, University of Southern California, Los Angeles, CA 90089-0042, USA
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Abstract. The Koopmans and Beckmann assignment model approach to problems of urban economics has led to considerable discussion. In our recent papers some of the issues raised were reviewed and also extensions that resolve many of the dilemmas suggested by the original model were proposed. Most of the approaches surveyed in our previous paper make little use of the duality properties of the assignment model. In the present paper it is argued that, under certain circumstances, dual variables interpretable as site rents and other dual variables that suggest the values of transferable development rights (TDRs) appear with model solutions. This view enhances the importance of assignment models to city planners and sheds light on how planners can guide optimal development along with the implementation of TDR policies.

> "Few things are free anymore—not even air. Paying for air is nothing new to downtown developers, though. Most of L.A.'s Central Business District is zoned to allow no more than 12 square feet of usable office space for every square foot of land. This makes building a 50-story tower impossible without collecting extra development rights—usually from a low-rise property owner with rights to spare and no plans to build a taller building.
>
> "Getting these rights has long been a tedious and expensive process, and new rules governing such exchanges have developers worried about how to pull off such deals in the future. Besides money and patience, builders downtown need the approval of the city's Community Redevelopment Agency, Planning Commission and City Council. Unless all three sign off on an exchange of air rights, the deal dies."
>
> Ron Galperin, in "A fight for rights to build skyscrapers", 1989, page 1

1 Introduction

Ever more of the world's cities are being planned in ways that seem to allow (and rely on) markets for development rights. Economists would expect efficiency gains over traditional ad hoc zoning plans because markets exploit information readily available to participants. Development rights can be more productively combined with complementary inputs if not locked to a site. They could be transferred to a more efficient location. Further, as their prices reflect their value they would be combined with nonlocational inputs in more efficient proportions. Or, when combined with a zoning plan, transferable development rights (TDRs) can be a tool, "to overcome the windfall-wipeout dilemma and the perverse economic incentives created by traditional zoning" (Barrows and Prenguber, 1975, page 549).

In either case, and as the introductory quotation shows, because a public plan is at stake, exchanges will usually take place on (allegedly) managed markets. Can planners manage TDR markets to optimize the city's development? If so, how?

Urban economists have treated TDR markets in the context of spatial equilibrium models (Carpenter and Heffley, 1981). Ghosh (1986) has applied such models to policy studies. We suggest a different modeling approach that we claim is more useful to TDR planners.

In an earlier paper (Gordon and Moore, 1989) we suggested that a planning model (hereafter, GM) should:
1. confront the simultaneity between markets for urban land and transportation services;
2. accommodate the reality of cities as places where externalities and common properties abound;
3. emulate the intertemporal albeit bounded nature of planning and decisionmaking;
4. fully exploit principles of economic theory; and
5. offer operational computability.

We presented a model that met all five objectives.

The purpose in this paper is to demonstrate that this GM model has duality properties that make it useful for optimal TDR-based planning. We will show that:
(1) if TDRs replace zoning, planners can use the dual variable information provided by GM to implement efficient land uses, thus providing an important operational link between land-use planning theory and practice;
(2) if TDRs are used to mitigate the windfalls and wipeouts created by a particular zoning plan, GM solves the planner's information problem, because the values of dual variables identify winners and losers;
(3) the value of wipeouts must necessarily exceed the value of windfalls, and additional public outlays are needed if constrained land-owners are to be completely compensated; and
(4) rents accruing to capital are also affected by zoning plans in ways ignored in the current literature. Unlike the neoclassical approach to TDR discussions, GM reveals that rental gains and losses may accrue to the owners of both land and capital.

2 Assignment models of urban structure

Versions of the assignment problem allow the matching of plants (or, more generally, any land-using activities that might locate in cities) with an equal number of available sites, given that the profitability of each location is different for each activity. Conventionally, this weighted bipartite matching is formulated as a simple linear program:

$$\text{maximize} \sum_{i,m} a_{im} X_{im} , \tag{1}$$

subject to

$$\sum_{m} X_{im} = 1, \qquad \forall i \in I, \quad \text{dual variable } q_i , \tag{2}$$

$$\sum_{i} X_{im} = 1, \qquad \forall m \in M, \quad \text{dual variable } r_m , \tag{3}$$

$$X_{im} = 0, 1, \qquad \forall i \in I, \quad m \in M , \tag{4}$$

where a_{im} is the profitability of site m for activity i; and X_{im} is an endogenous, binary variable equal to one if activity i is assigned to location m and is equal to zero otherwise. Constraint (2) ensures that plant i is located at exactly one site, constraint (3) ensures that site m receives exactly one plant, and q_i and r_m are the dual variables associated with constraints (2) and (3), respectively.

The fact that the right-hand side of the constraint set consists of a vector of ones identifies this formulation as the assignment version of the transportation problem, and thus constraint (4) is redundant. This is a unimodular linear program subject to combinatorially efficient solution procedures. As Koopmans and Beckmann (1957) pointed out, the assignment linear program is a very restrictive formulation that captures only some aspects of urban location. The GM model (below) relaxes most of these restrictions.

The dual formulation of the simple assignment model remains valid for the GM elaboration:

$$\text{minimize} \sum_i q_i + \sum_m r_m \,, \tag{5}$$

subject to

$$q_i + r_m \geqslant a_{im}, \qquad \forall i \in I, \quad \forall m \in M, \quad \text{dual variable } X_{im} \,, \tag{6}$$

$$q_i, \, r_m \text{ unrestricted in sign}, \qquad \forall i \in I, \quad \forall m \in M. \tag{7}$$

Koopmans and Beckmann (1957) identified q_i and r_m as 'plant rents' and 'site rents', respectively. This is a plausible interpretation, because at the optimum,

$$q_i + r_m = a_{im}, \qquad \text{for } X_{im} = 1 \,, \tag{9}$$

and

$$q_i + r_m \geqslant a_{im}, \qquad \text{for } X_{im} = 0 \,. \tag{10}$$

Thus, the optimal configuration implies that there is no incentive for a locator to chose another location. Plant rents accrue to the owners of the mobile factor (capital) and site rents accrue to the owners of the immobile factor (land). Lind (1973) showed that the standard Wingo (1961), Alonso (1964), Muth (1969), Mills (1972) model of urban land use is really a special case of the assignment model in which there are large numbers of bidders and competition of such intensity that all plant rents must be bid for sites. In the small auction case, above, it is sufficient for owners of capital to outbid a next-highest bidder for a given site and to retain any remaining profitability.

The example included in the Koopmans and Beckmann paper is instructive. They defined a (4×4) matrix of profitabilities, $\mathbf{A}$, the values of which are

Plant, i	Location, m				q_i
	1	2	3	4	
1	25	20*	5	19	10
2	18	3	0*	12	3
3	22*	4	2	12	6
4	16	7	−2	10*	1
r_m	16	10	−3	9	

The optimal vector assignment is indicated by asterisks. Note that, at the optimum, $\boldsymbol{q} = (10\;3\;6\;1)$, $\boldsymbol{r} = (16\;10\;-3\;9)$, and

$$X_{31} = 1, \quad q_3 + r_1 = 6 + 16 = 22 = a_{31} \,;$$

$$X_{12} = 1, \quad q_1 + r_2 = 10 + 10 = 20 = a_{12} \,;$$

$$X_{23} = 1, \quad q_2 + r_3 = 3 - 3 = 0 = a_{23} \,;$$

$$X_{44} = 1, \quad q_4 + r_4 = 1 + 9 = 10 = a_{44} \,.$$

All other X_{im} are equal to zero and all other dual constraints hold with strict inequality. Clearly, this model implies that profitabilities are divided as rental income between the owners of plants and sites.

Koopmans and Beckmann also appended quadratic terms to the objective function to account for economic interdependencies between activities, and demonstrated the consequent instabilities that plague static quadratic assignment models. That is, if interdependencies are included, the optimal solution will not, in general, be integral. GM incorporates an intertemporal approach that avoids these difficulties.

Adding time subscripts to the previous notation, GM optimizes the following objective function:

$$\text{maximize} \sum_{i,m} \{a_{imt} - R_i[1 - X_{im(t-1)}]\} X_{imt} , \tag{11}$$

subject to constraints (2) through (4). $\boldsymbol{R}$ is a vector of exogenous relocation costs that are (for simplicity) independent of the distance moved.

Planners could maximize expression (11) separately for each time period if **A**, the matrix of (present-value) profitabilities, changed in any way between periods. How would the matrix **A** change? GM places the following updating function outside the optimization:

$$a_{im(t+1)} = a_{im0} + \sum_{i,m} (b_{imjn} - f_{ij} d_{mn}) Y_{imjnt} , \tag{12}$$

$$b_{imjn} = e_{ij}(d_{mn}) = \begin{cases} e_{ij}, & \text{if } d_{mn} \leqslant K_{ij} , \\ 0 , & \text{if } d_{mn} > K_{ij} , \end{cases} \tag{13}$$

where

Y_{imjnt} is equal to $X_{imt} X_{jnt}$,
e_{ij} is the potential externality effect of activity j on activity i,
d_{mn} is the distance between sites m and n,
f_{ij} is the interactivity flows between activities i and j,
K_{ij} is a spatial proximity threshold, and
b_{imjn} is an $(N^2 \times N^2)$ matrix that inventories the externalities realized if and only if activity pairs locate at sites that are closer than threshold K_{ij}.

The updating function is premised on the assumption that all benefits (including those that depend on the location of other activities) known to accrue to activity i at site m in period t will be included in the (present-value) bid made by the operator of plant i for location m during the next period. Each bidder makes a ceteris paribus evaluation of the locational patterns that were observed in the previous period. Each time period is defined by the arrival (or departure) of exactly one new bidder, and thus all of the information needed for the determination of accurate bids is available. A pure assignment problem can be defined for every time period, and the instability problems of the static Koopmans and Beckmann problem do not arise. If discount factors are adjusted, time periods need not be of equal length.

Expression (11) suggests that new locational advantages and disadvantages must be traded off against movement costs, which could also change over time in response to changing technologies and/or regulations. The GM approach also includes endogenous transportation costs and flows. Ideally, this might imply the following model needs to be solved for each time period:

$$\text{maximize} \sum_{i,m} a_{im} X_{im} + \sum_{i,m,j,n} (b_{imjn} - f_{ij} d_{mn}) Y_{imjn} - \sum_{k} \int_0^{f_k} c_k(w) \mathrm{d}w , \tag{14}$$

subject to

$$\sum_{i} X_{im} = 1, \qquad \forall m \in M , \tag{15}$$

$$\sum_{m} X_{im} = 1, \qquad \forall i \in I , \tag{16}$$

and

$$Y_{imjn} = X_{im}X_{jn}, \qquad \forall i, j \in I, \quad \forall m, n \in M , \tag{17}$$

$$\sum_{s,m,n} Y_{imjn} f_{simjn} = f_{ij}, \qquad \forall i, j \in I , \tag{18}$$

$$\sum_{s,i,m,j,n} Y_{imjn} \partial_{ksmn} f_{simjn} = f_k, \qquad \forall k \in K , \tag{19}$$

$$f_{simjn} \geqslant 0, \qquad \forall s \in S, \quad \forall i, j \in I, \quad \forall m, n \in M , \tag{20}$$

$$X_{im} = 0, 1, \qquad \forall i \in I, \quad \forall m \in M , \tag{21}$$

where
$c_k(.)$ is an increasing (average) cost function of the flow on link k,
f_k is the total flow on congestable transportation link k,
f_{simjn} is the flow on path s from activity i located at site m to activity j located at site n, and
∂_{ksmn} is a binary indicator equal to one if link k is on path s from site m to site n, and is equal to zero otherwise.

The sum of integrals in the objective function captures user-equilibrium transportation costs accruing on the transportation network. Constraints (18) and (19) are the conventional path-flow constraints for static network equilibrium. Constraint (18) ensures that the traffic flows between activity i located at site m and activity j located at site n over all paths s satisfies the exogenous interaction requirement for activities i and j. Constraint (19) ensures that the flows on all of the paths in which link k is used contribute to the total flow on the link.

The solution to the formulation expressed by equations (14)-(21) is subject to the price sustainability questions visited on the solution to the original Koopmans-Beckmann problem. It is not even clear that this formulation has a unique optimum, because expression (18) is cubic. Last, this path-flow formulation requires the preenumeration of all network paths, a combinatorially expensive prospect.

Alternatively, the GM model offers a simplified iterative procedure that meets these difficulties (figure 1). The simplified iterative procedure combines (separable) solutions to a pure assignment problem and a nonlinear equilibrium path-flow problem. Given changes in equilibrium path costs, external effects, and known relocation costs, locators are assumed to update site bids under the assumption that all other locators will retain their current sites. These ceteris paribus updates produce new site allocations, and bids continue to be adjusted. Computationally, the most expensive step in this process is the solution to a convex nonlinear programming problem with linear constraints (Moore and Gordon, 1990).

For updating of the elements of $\mathbf{A}$, equation (12) is a key operation. Initial values of $\mathbf{A}$ have been presumed. Actually, elements of $\mathbf{A}$ summarize information on the supply and demand for site characteristics. If there are I activities, M sites, and H site characteristics, then the matrix of profitabilities, $\mathbf{A}$, can be conceptualized as the product of $\mathbf{A}_1$, dimension $I \times H$, which lists demands of each activity for each characteristic, and $\mathbf{A}_2$, dimension $H \times M$, describing the supply of characteristics (including accessibility to exogenous and fixed sites, such as the coastline) at each site. It is logical that the 'semi-net revenue' any activity associates with any site is a function of the demand and supply of characteristics. As the elements of $\mathbf{A}$ are denominated in dollars, it follows that elements of $\mathbf{A}_2$ are measured in physical terms (buildable square feet, days per year of clean air, miles to the coast, degree of

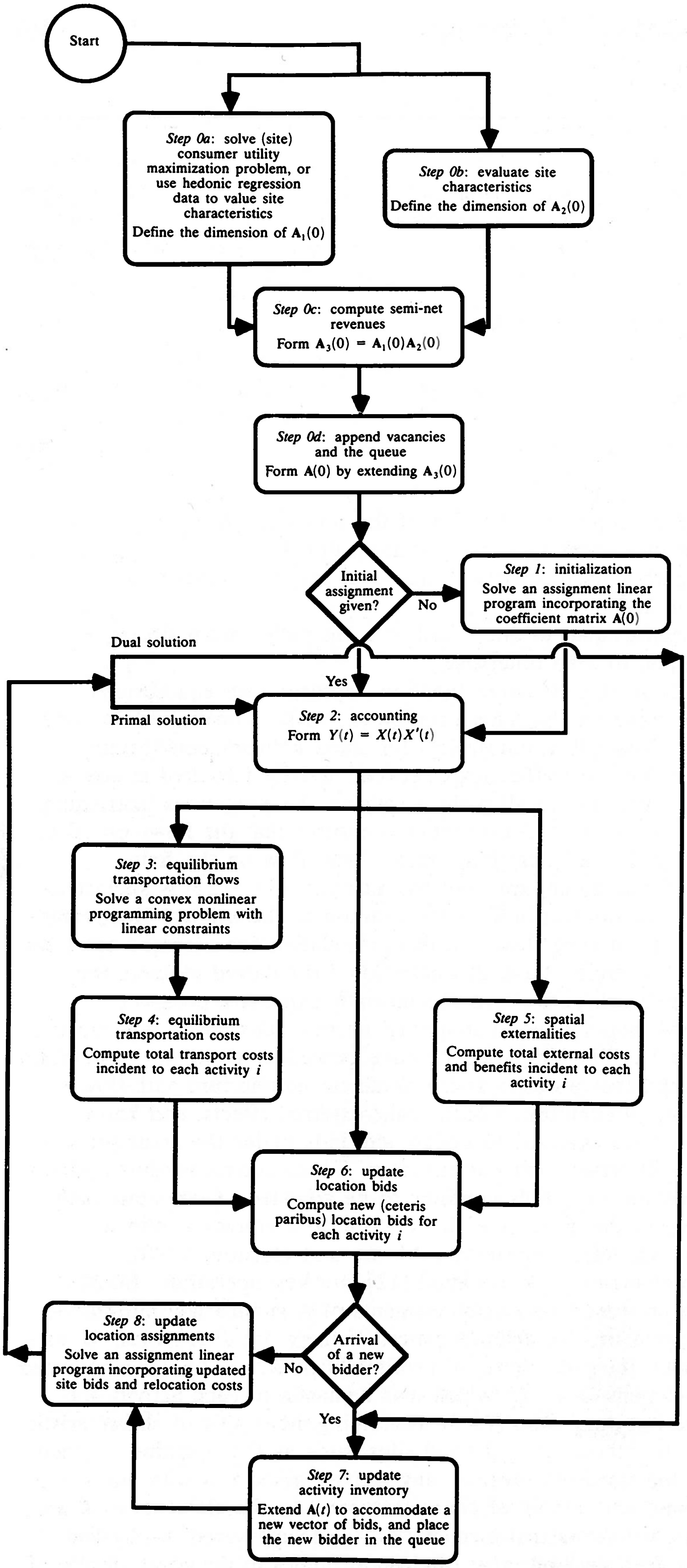

Figure 1. Algorithmic representation of model GM.

slope, etc), and elements of $\mathbf{A}_1$ are in dollars or willingness-to-pay for each unit of attribute. In other words, $\mathbf{A}_1$ summarizes demand information that is obtained from standard consumer utility maximization. In this case, there would be I consumer groups and H characteristics. Utility maximization would be over characteristics rather than commodities, as in the theory of Lancaster (1966).

3 Duality and development

As figure 1 indicates, the duality properties of the iterative model are the same as those shown in the numerical example above. They demonstrate that plant rents are likely to be high for activity types that are scarce. Welfare would increase if such restrictions were relaxed, that is, if more development rights for the high-plant-rent activities were made available.

Planners could expand development rights optimally by using the model. A key aspect of GM is the arrival of new activities (bidders for sites) in each period. The most meaningful way to treat this phenomenon is to augment $\mathbf{A}$, the matrix of profitabilities, by appending one row that corresponds to a null activity called 'vacancy', and one column that corresponds to a null location, or queue. The vacancy activity bids nothing for sites and can be simultaneously assigned to any number of locations. Activities bid nothing for access to the queue, and there is no constraint on the number of activities that can locate there simultaneously. When nonvacancy activities offer positive bids for sites, vacancies are displaced. Vacancies displace unprofitable activities. The market relegates all unsuccessful bidders to the null site, that is, to a profitless queue. These unprofitable activities escape losses by leaving the city, returning only when new conditions enable them to compete.

The augmented matrix $\mathbf{A}$ that reflects this approach for a system of I bidding activities and M physical sites is dimensioned as follows:

Bidding activities, i	Physical sites, m							Null site (queue), $M+1$
	1	2	⋯	m	⋯	$M-1$	M	
1	$a_{1,1}$	$a_{1,2}$	⋯	$a_{1,m}$	⋯	$a_{1,M-1}$	$a_{1,M}$	0
2	$a_{2,1}$	$a_{2,2}$	⋯	$a_{2,m}$	⋯	$a_{2,M-1}$	$a_{2,M}$	0
⋮	⋮	⋮		⋮		⋮	⋮	⋮
i	$a_{i,1}$	a_{i2}	⋯	$a_{i,m}$	⋯	$a_{i,M-1}$	$a_{i,M}$	0
⋮	⋮	⋮		⋮		⋮	⋮	⋮
$I-1$	$a_{I-1,1}$	$a_{I-1,2}$	⋯	$a_{I-1,m}$	⋯	$a_{I-1,M-1}$	$a_{I-1,M}$	0
I	$a_{I,1}$	$a_{I,2}$	⋯	$a_{I,m}$	⋯	$a_{I,M-1}$	$a_{I,M}$	0
Null activity (vacancies)								
$I+1$	0	0	⋯	0	⋯	0	0	0

Were the growth of the city to be simulated (and/or guided) in this way, could the contents of the queue be updated intelligently? Inspection of dual variables associated with activities assigned to sites would identify the most profitable new bidders. New locators are not necessarily synonymous with new bidders: not all bidders are able to obtain sites. New bidders differ from existing bidders in that the latter have been previously located either at physical sites or in the queue.

4 Land markets and externalities

The simplest externality problems are resolved if markets are permitted to operate. Governments are required to award property rights and enforce contracts. The simplest examples usually exclude transactions costs by involving just two agents.

If the agents are proximate, sharing an enclosed room or local airshed, if one of them wants to smoke whereas the other prefers to breathe clean air, they can reach an efficient outcome if they bargain in light of assigned property rights. According to this well-known Coasian (Coase, 1960) example, the right to smoke may be assigned, prompting the clean-air preferrer to buy reduced smoke from the smoker. Or, the right to clean air may be enforced in which case the smoker must buy permission to smoke. Identical smoking (or densities, or development) arise in either regime. Though efficient allocation results in both cases, the different rights assignments have opposite distributional implications.

Consider locational choice. Under either rights regime, bidders either stay where they are or they move to other sites, in light of land rents, conditions elsewhere, and the costs of moving. As an agent's welfare at any location is affected by rights, relocation decisions are guided by the rights regime that locators are subject to at the original location. In this case, there is less symmetry than Coase suggested. All of this is even more apparent when many interdependent locators are considered, as in the case of a city. By using GM to represent the city, we see that the value of the optimand would likely change with different rights assignments, revealing that the efficiency consequences of any agent's relocation can be considerable.

Following the logic of GM further, we see that the updating function that captures the anticipations (and the bid formations) of agents provides the key. The rights regime that is enforced, if known, enters the anticipations of agents. They are expected to know just how much of a (technologically determined) potential spillover or spillovers will accrue at sites; they also know what contracting arrangements (owing to rights assignments) are to be expected. The model, then, handles rights regimes and allows us to study the consequences of various TDR discussions.

Many approaches to land-use planning have been suggested (Hagman and Misczynski, 1978). For the case of zoning, efficiency or other (for example, historical preservation, 'equity', aesthetic) targets may be at stake. The available literature on the TDR approach to planning is similarly diverse. TDR markets can replace zoning if efficient allocations are desired. Alternatively, TDRs used in association with zoning have been suggested as a way to mitigate 'windfalls and wipeouts'.

We do not suggest that the allocations identified in the primal solution of the model be used to dictate land uses and attendent development decisions. That course would transfer development rights to the public sector and obviate any market exchange of rights. Still, planners seeking to replace zoning with a TDR-based system of land-use control can use the model in a variety of ways. First, if planners believe that the model simulates land markets, then they can use it as a forecasting device (Moore and Gordon, 1991). Or, if planners expect that markets for development rights (as opposed to land-ownership rights) are sufficiently imperfect to warrent intervention, they can implement optimal land-use patterns by selling development rights. For each sale in each period, the planner's reservation price for permitting each development activity is given by the optimal values of the vector $\boldsymbol{q}$. The natural definition of a period is an update of the $\mathbf{A}$ matrix (Gordon and Moore, 1989), but a more institutional definition of temporal development rights is certainly workable. In Koopmans and Beckmann's one-period (4×4) example, four development rights auctions would be executed with $\boldsymbol{q} = (10\ 3\ 6\ 1)$. Successful bidders will have no profitable alternative but to develop sites as indicated by the primal solution. Expenditures on development rights, optimal locations, and an optimal quantity of capital leaves the developer with zero profits. Suboptimal land uses would create incentives for mutually advantageous exchanges. Alternatively, successful bidders could sell their development rights to other agents who would face the same incentives.

In any period, dual values for plant rents of *excluded* activities indicate what the corresponding development rights would sell for. Planners could auction development rights for the activity at the head of the queue at the reservation price indicated by the dual solution. This would be the efficient level for that activity during the period. Planners would be assured that markets would guide these rights to the best site. The right would most likely be exercised at the site that would generate revenues sufficient to permit purchase of the right.

Alternatively, planners can use TDRs and zoning simultaneously. They usually do so in an attempt to counter the windfalls and wipeouts that accompany many zoning plans. Yet, in the literature it is pointed out that there is considerable difficulty in synchronizing these quantities. Barrows and Prenguber (1975) report that,

> "obviously, the DR market must function in a reasonable manner in order for restricted owners to be compensated. The demand side of the DR market may be quite unpredictable. A highly sophisticated planning agency will be required not only to produce the land use plan but also to accurately forecast development demand in the area. A strong demand for development in the development zone will be required to ensure adequate DR demand so that DR will be transferred and restricted owners compensated" (page 552).

And,

> "the amount of downzoning would have to be carefully synchronized with anticipated development (and DR) demand in order to ensure compensation to restricted land owners" (page 550).

Efficiency is probably a secondary concern when new activity is to be guided from designated 'conservation' zones to designated 'development' zones. If TDRs are to be used to mitigate windfalls and wipeouts resulting from such interventions, how many rights should planners allow conservation zone owners to sell? Even if each owner's loss is known, how do planners know that the TDR quantities and prices that would clear the market would reach this figure? Would the same figure balance the windfalls that the plan has created? The three values would equate only by the sheerest coincidence.

We can use GM to resolve these problems by following a slightly different approach. Assume the system is near the optimum identified by GM. Next, consider a GM solution with the added constraint that there be a conservation zone. This new constraint can only decrease the optimal value of the objective function, Z, by some quantity ΔZ. The new solution will redistribute activities away from the conservation zone in an optimal manner. No single development zone would have to be identified by planners. If it is assumed that plant-rent changes are negligible (an assumption we will shortly relax), land-rent values from the new dual solution would indicate the value of windfalls that were created by the constraint that there be a conservation zone. Yet, the reduced value of the objective function indicates what we have already noted—that there is not enough value in the system for these windfalls to compensate the wipeout (ΔZ = windfalls − wipeouts, where ΔZ is a negative number). Planners could award conservation-zone owners the right to sell TDRs with a value equal to the sum of windfalls, knowing that this much demand would be forthcoming. If they deem this to be inequitable, they could supplement these revenues with grants equal to ΔZ. Again, the solution of GM supplies the data.

Whereas all rents are assumed to accrue to land-owners in neoclassical analysis, GM provides this result only as a special case. It is entirely plausible for some rents to accrue to the owners of certain types of capital. For this reason, the dual values GM identifies as plant rents merit attention. Moreover, simulation experiments with plausible parameter values demonstrate that plant-rent values can be substantially suppressed by the imposition of a conservation zone (or zones). Under reasonably general conditions, the pattern of windfalls and wipeouts is therefore much more complicated than suggested in the literature (Moore, 1991).

5 Conclusions

We have specified a computable general equilibrium model that captures the dynamic character of urban structure and highlighted the information provided by the dual conditions of the model. These contribute to recent discussions concerning the role of TDR use in land-use planning. In particular, we have shown that:

1. if TDRs replace zoning, dual variable information can be used by planners to implement efficient land uses;
2. if TDRs are used to mitigate the windfalls and wipeouts created by a particular zoning plan, GM is needed to solve the planner's information problem;
3. the value of wipeouts must exceed the value of windfalls; and
4. rents accruing to capital are also affected by zoning plans: GM shows how.

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